

*Attorney's Docket No. TN273*  
*Amendment*

*Serial No. 10/647,826*  
*21 September 2006*

**REMARKS**

Claims 1-25 and 30-33 are pending in the instant application. Claims 1-25 are allowed. Claims 30-33 stand rejected under 35 U.S.C. 112, second paragraph.

**REJECTIONS UNDER 35 U.S.C. 112, SECOND PARAGRAPH**

In paragraph 1 of the Office Action dated November 30, 2006 on p. 2, the Examiner asserts that claims 30-33 stand rejected under 35 U.S.C. 112, second paragraph. In asserting this rejection, the Examiner stated in part:

"Claim 30 recites, 'a set of at least one processing modules for performing programmable processing tasks defined within the set of processing modules.' The Examiner is unsure at to what 'a set of at least one processing modules ...' is a set of consists of two or more, so the Examiner is unclear of what Applicant is trying to state with this limitation concerning the number or arrangement of processing modules because the specification states processors and multiprocessors." (EMPHASIS ADDED)

The Examiner seems to be asserting that under no circumstances can a set as commonly understood in the art could include only one element. In response, the Applicants maintain a set as commonly understood in the art may include any number of elements, including both zero elements (which is commonly understood to be "an empty set" or "a null set") and one element. In support of this understanding, the Applicants submit pp. 16-17, entitled "CONCEPTS OF ALGEBRA - Algebra of Sets" from the CRC Standard Mathematical Tables, 27<sup>th</sup> Edition, 1984, in which algebra of sets includes the empty set and general notations of set having any number of elements. As such, the above quoted limitation of a set of at least one processing modules recites the use of one or more processing modules acting together to perform the recited functions.

With respect to the additional assertion that the limitations of "the set of processing modules" as recited is indefinite, the Applicant respectfully maintains that the set of processing modules refers to the set discussed above. If the Examiner asserts that this common use of the term "the set" to refer to a previously recited set is indefinite, Applicants

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respectfully request that the Examiner contact the undersigned counsel to assist in defining an acceptable claim limitation for reciting this reference that would be acceptable to the Examiner.

With respect to the limitation of "a set of system processing modules" that the Examiner also asserts is indefinite, Applicants maintain that this language is also proper. This limitation refers to a set of system processing modules which are to be considered separate from the above mentioned set of processing modules where these system processing modules are used "for booting said computer system and launching processing tasks associated with the above-mentioned set of processing modules. Once again, Applicants respectfully request that the Examiner contact the undersigned counsel to assist in defining an acceptable claim limitation for reciting this reference that would be acceptable to the Examiner if the Examiner asserts that this common use of the term "a set" to refer to a different set of modules is indefinite.

In light of these arguments, it is believed that the rejection based in Section 112 is overcome.

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**CONCLUSION**

Based on all these considerations and amendment, the applicant respectfully requests reconsideration and allowance of the claims. If any issues remain that preclude issuance of this application, the Examiner is again urged to contact the undersigned attorney.

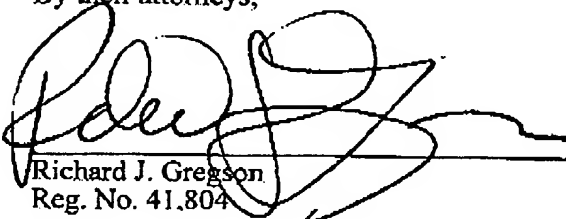
Respectfully Submitted,

DIANE L. KESSLER, ET AL.

By their attorneys,

Date: 21 Feb 2007

By

  
Richard J. Gregson  
Reg. No. 41,804

UNISYS CORPORATION  
Unisys Way, MS/E8-114  
Blue Bell, PA 19424  
Phone (215) 986-3325  
Fax (215) 986-3090

# CRC Standard Mathematical Tables

27th Edition

Editor of Mathematics and Statistics

**William H. Beyer, Ph.D.**

Professor of Mathematics and Statistics  
and Head of the Department of Mathematics  
and Statistics, University of Akron, Akron, Ohio



CRC Press, Inc.  
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## PREFACE

The rapid development of applied mathematics, statistics, computer technology, as well as advances in the physical and engineering sciences, and the widespread availability and use of hand-held calculators and microcomputers have certainly had their effect on the nature of published reference material. In spite of these advances, it is clear that there is still universal acceptance and satisfaction with the base of reference material that has been provided in handbooks published by CRC Press, Inc. It has long been the established policy of CRC Press, Inc. to publish in handbook form, the most up-to-date, authoritative, and logically arranged reference material available. This 27th Edition of the *CRC Standard Math Tables* continues to follow this policy.

The improvement in this edition is dictated only by the desire to make it an important aid to the teaching profession, to the student, and to the many others who require the table or fact for investigating and creating answers to today's challenging problems. The material is presented in a multi-sectional format, with each section containing a valuable collection of fundamental reference material both expository and tabular — necessary for use in today's world. The customary reference data contained in earlier editions, plus the new expository and tabular material included in the 25th and 26th Editions, is repeated. However, several desirable additions — expanded sections on numerical differentiation and integration; new material on numerical solutions to ordinary differential equations; material on analysis-of-variance — are to be noted throughout this new edition. Tables involving trigonometric, exponential, logarithmic, and hyperbolic functions have been reworked — some have been omitted some have been greatly reduced in size. It is hoped that the changes will prove to be beneficial to the users of the handbook. It is suggested that if the user desires more extensive and/or additional reference material than is provided in this edition, he or she refer to other CRC publications such as the *CRC Handbook of Tables for Mathematics* (renamed *CRC Handbook of Mathematical Sciences*), the *CRC Handbook of Tables for Probability and Statistics*, etc.

The Editor gratefully acknowledges the services rendered by Paul Gottlieb, Senior Editor, for the handling of the detail work which is so essential in the final production of this edition.

As in the past, CRC Press, Inc. and the Editor invite and welcome constructive comments from the many users of the handbook. These comments are a most effective means for keeping the editions of the handbook updated and abreast of the times.

William H. Beyer, Editor

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## ALGEBRA

## BASIC CONCEPTS IN ALGEBRA

DR. W. E. DESKINS

## I. ALGEBRA OF SETS

Intuitively a set is a collection of objects called the elements of the set. Set and set membership are generally accepted as basic, undefined terms used to define and construct mathematical systems.

The notation  $a \in A$  indicates that  $a$  is an element of the set  $A$ . The notation  $a \notin A$  means that  $a$  is not a member of  $A$ .

A set is sometimes specified by listing its elements within a set of braces:  $\{a\}$  is the set containing only the element  $a$ .

Set  $A$  is a subset of set  $B$  provided  $a \in A$  implies  $a \in B$ . This is denoted by  $A \subseteq B$ . Every set has as a subset the empty or null set, denoted by  $\phi$ , which has no elements.

Set  $A$  equals set  $B$ , written  $A = B$ , if and only if  $A \subseteq B$  and  $B \subseteq A$ .  $A$  is a proper subset of  $B$ , sometimes indicated by  $A \subset B$ , if and only if  $A \subseteq B$  and  $A \neq B$ ; then  $B$  has at least one element which does not belong to  $A$ .

The Cartesian product of sets  $A$  and  $B$ , denoted by  $A \times B$ , is the set of all ordered pairs  $(a, b)$  where  $a \in A$  and  $b \in B$ . A subset  $R$  of  $A \times A$  is a binary relation on  $A$ , and this is an equivalence relation on  $A$  provided (i)  $(a, a) \in R$  for every  $a \in A$ , (ii)  $(a, b) \in R$  implies  $(b, a) \in R$ , and (iii)  $(a, b) \in R$  and  $(b, c) \in R$  imply  $(a, c) \in R$ .

Ordinary equality of numbers, equality of sets, and congruence of plane figures are examples of equivalence relations.

A subset  $F$  of  $A \times B$  is a *function* from  $A$  to  $B$  provided each element of  $A$  appears exactly once as the first element of a pair in  $F$ . A function  $F$  from  $A$  to  $B$  is *onto* provided each element of  $B$  appears at least once as the second element of a pair in  $F$ . It is *one-to-one* provided each element of  $B$  appears at most once as the second element of a pair in  $F$ . A function from  $A \times A$  to  $A$  is a *binary operation* on  $A$ .

Addition and multiplication of ordinary numbers are examples of binary operations. If consideration is restricted to elements and subsets of a particular set  $I$ , then  $I$  is the universal set.

Common binary operations on subsets of  $I$  are:  $A \cup B$ , the union or join of sets  $A$  and  $B$ , is the set of all elements of  $I$  which belong to either  $A$  or  $B$  or both  $A$  and  $B$ .

$A \cap B$ , the intersection or meet of sets  $A$  and  $B$ , is the set of all elements of  $I$  which belong to both  $A$  and  $B$ .

$A \setminus B$ , the difference of sets  $A$  and  $B$ , is the set of elements of  $I$  which belong to  $A$  but not  $B$ .

The difference  $A \setminus A$  is denoted by  $A'$  and called the complement of  $A$  (relative to  $I$ ). Except in dealing with the concept of complementation the use of a universal set is not essential to the above ideas.

Some theorems basic to the Algebra of Sets:

Let  $A$ ,  $B$ , and  $C$  be arbitrary subsets of a universal set  $I$ .

- (a) (Commutativity)  $A \cup B = B \cup A$  and  $A \cap B = B \cap A$ .  
 (b) (Associativity)  $(A \cup B) \cup C = A \cup (B \cup C)$  and  $(A \cap B) \cap C = A \cap (B \cap C)$ .

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- (c) (Distributivity)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  and  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

- (d) (Idempotency)  $A \cup A = A$  and  $A \cap A = A$ .

- (e) Properties of  $I$  and  $\phi$ :  $A \cap I = A$  and  $A \cup \phi = A$ .

- $A \cup I = I$ , and  $A \cap \phi = \phi$ .

- (f)  $(A \cap B) \cup (A \cap B) = A$ .

- (g)  $(A \cap B) \cup B = A \cup B$ .

- (h)  $A \subseteq A \cup B$ .

- (i)  $A \cap B \subseteq A$ .

- (j)  $A \cup B = A$  if and only if  $B \subseteq A$ .

- (k)  $A \cap B = A$  if and only if  $A \subseteq B$ .

- (m)  $A \cap B = A(A \cap B)$ .

- (n) (DeMorgan's Theorem)  $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$  and  $(A \cap B) \cup (A \cap C) = A \cap (B \cap C)$ .

- (o)  $(A \cup B)' = A' \cap B'$  and  $(A \cap B)' = A' \cup B'$ .

- (p)  $A \cup A' = I$  and  $A \cap A' = \phi$ .

9. A mathematical system  $S$  is a set  $S = \{E, O, A\}$  where  $E$  is a nonempty set of elements,  $O$  is a set of relations and operations on  $E$ , and  $A$  is a set of axioms, postulates, or assumptions concerning the elements of  $E$  and  $O$ .

10. The Algebra of Sets provides an example of a mathematical system called a Boolean Algebra (or Boolean Ring) which is defined as:

- Set  $E$  of elements  $a, b, c, \dots$ ;

- Set  $O$  of 2 binary operations  $\oplus$  and  $\otimes$ ; (Here  $a \oplus b$  denotes the image of  $(a, b)$  under the binary operation.)

- Set  $A$  of axioms for all  $a, b, c$  of  $E$ :

- A<sub>1</sub>. The binary operations are commutative; i.e.,

- $a \oplus b = b \oplus a$  and  $a \otimes b = b \otimes a$ .

- A<sub>2</sub>. Each binary operation is distributive over the other; i.e.,

- $a \oplus (b \otimes c) = (a \oplus b) \otimes (a \oplus c)$  and  $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$ .

- A<sub>3</sub>. There exist elements  $e$  and  $z$  in  $E$  such that for each  $a \in E$ ,  $a \oplus e = a$  and  $a \otimes z = a$ .

- A<sub>4</sub>. For each  $a \in E$  there exists an element  $a' \in E$  such that  $a \oplus a' = e$  and  $a \otimes a' = z$ .

- In the algebra of subsets of a (universal) set  $I$ ,  $\phi$  plays the role of  $z$ ,  $I$  that of  $e$ ,  $\cup$  that of  $\oplus$ , and  $\cap$  that of  $\otimes$ .

11. A Boolean Algebra has the Principle of Duality: If the interchanges of  $\oplus$  and  $\otimes$  are made in a correct statement, then the result is also a correct statement.

12. In addition to the Algebra of Sets which is a Boolean Algebra, other representations of Boolean Algebra that are interesting of themselves and valuable for their applications are:

- (a) The Algebra of Symbolic Logic

- (b) The Algebra of Switching Currents